

Structural Studies on the Rare Earth Carboxylates

6. A Reinvestigation of Trisodium-tris(oxydiacetato)lanthanoidate(III)

Di(sodium Perchlorate) Hexahydrate

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The unit cell dimensions of the isostructural series of lanthanoid oxydiacetate compounds $\text{Na}_3[\text{M}(\text{C}_4\text{H}_4\text{O}_5)_3] \cdot 2\text{NaClO}_4 \cdot 6\text{H}_2\text{O}$, $\text{M}=\text{Ce}-\text{Lu}$, have been determined using powder photographs obtained with a Guinier focusing camera. The crystals are trigonal with the space group (No. 155) and with $Z=3$. Between the cerium and lutetium compounds the value of a decreases from $9.8117 \pm 0.0027 \text{ \AA}$ to $9.6787 \pm 0.0015 \text{ \AA}$ while the value of c decreases from $28.410 \pm 0.013 \text{ \AA}$ to $28.043 \pm 0.006 \text{ \AA}$. The previously determined atomic parameters of the neodymium and ytterbium compounds have been refined by least-squares methods using three-dimensional X-ray intensity data newly collected with an automatic single crystal diffractometer. The coordination polyhedron around the nine-coordinated lanthanoid ion is found to decrease $0.133 \pm 0.015 \text{ \AA}$ along the c axis and $0.137 \pm 0.018 \text{ \AA}$ along the a and b axes between the neodymium and ytterbium compounds. The contraction thus imposed on the unit cell dimensions is obstructed through the whole series by the other parts of the structure. For the heaviest lanthanoid ions this trend is reenforced by van der Waals contacts between the oxygen atoms coordinated to the lanthanoid ion.

The structure of the isomorphous neodymium, gadolinium, and ytterbium oxydiacetate (or diglycolate) compounds of the composition trisodium-tris(oxydiacetato)lanthanoidate(III) di(sodium perchlorate) hexahydrate, *i.e.*, $\text{Na}_3[\text{M}(\text{C}_4\text{H}_4\text{O}_5)_3] \cdot 2\text{NaClO}_4 \cdot 6\text{H}_2\text{O}$, has previously been determined by the present author¹ from three-dimensional intensity data recorded with the Weissenberg multiple film technique. Although the general features of the structure were obtained, the main purpose of that investigation was not reached: the dimensions of the coordination polyhedron around the lanthanoid ions could not be determined accurately enough to reveal the expected decrease in the order $\text{Nd} > \text{Gd} > \text{Yb}$. To get more accurate $\text{M}-\text{O}$ and $\text{O}-\text{O}$ distances than the "photographic" ones, the neodymium and ytterbium compounds,

referred to as NDG and YDG below, have been reinvestigated with an automatic single crystal diffractometer.

All lanthanoid compounds of the composition stated above have the same structure. The second aim of the present investigation is to study the variation of the unit cell dimensions in this isostructural series as a function of the crystal radius of the trivalent lanthanoid ion and to correlate the results with the now obtained single crystal measurements.

EXPERIMENTAL

Single crystal work. The compounds were prepared from the appropriate lanthanoid perchlorates and sodium oxydiacetate as described before.¹ Single crystals of NDG and YDG were mounted along their *b* axes and preliminary Weissenberg photographs were taken with $\text{CuK}\alpha$ radiation. The samples chosen for further investigation had a stout prismatic habit with the approximate dimensions $0.1 \times 0.2 \times 0.1 \text{ mm}^3$ and $0.2 \times 0.4 \times 0.2 \text{ mm}^3$ for NDG and YDG, respectively. They were elongated in the *b* direction.

The intensity data were collected with an automatic single crystal diffractometer of type PAILRED using $\text{MoK}\alpha$ radiation. The take-off angle was 6° . The radiation was made monochromatic by reflexion from the (200) plane of a lithium fluoride crystal. The data were collected by the equi-inclination and ω scan techniques. The scan rate was $1^\circ/\text{min}$ for 3° scan ranges. Stationary background counts, B_1 and B_2 , of 1 min were taken at each end of the scan interval. All measured backgrounds fulfilled the condition $0.5 < B_1/B_2 < 2.0$. A counter aperture size of 2° was used. Coincidence losses were negligible and the pulse height discrimination levels were set for approximately a 90 % window centered on the $\text{MoK}\alpha$ peak. As a check an electronic stability during the period of data collection the intensities of standard reflexions were measured at regular intervals. No systematic change in these standards was observed.

The intensities of all independent reflexions within the copper sphere ($\sin \theta < 0.4610$) were measured. The total number of independent reflexions collected was 698 for NDG and 683 for YDG. The corrected integrated peak counts I were calculated from the eqn.

$$I = C - t_c(B_1 + B_2)/2t_b$$

where C is the total integrated peak count obtained in time t_c , and t_b is the time for each of the background counts. The corrected intensities were assigned standard deviations according to the formula

$$\sigma(I) = [C + t_c^2(B_1 + B_2)/4t_b^2 + (kI)^2]^{1/2}$$

The value of k was selected as 0.03. The effect of the term $(kI)^2$ is to weight down strong reflexions.

The values of I and $\sigma(I)$ were corrected for Lorentz, polarization, and absorption factors. The linear absorption coefficient μ is 20.2 cm^{-1} for NDG and 34.2 cm^{-1} for YDG. The transmission coefficients, evaluated by numerical integration, were found to range from 0.83 to 0.88 for the sample of NDG and from 0.52 to 0.59 for the sample of YDG. No corrections for extinction effects were applied.

Powder work. Powder photographs were taken at 20°C with $\text{CuK}\alpha$ radiation ($\lambda = 1.5418 \text{ \AA}$) in a Guinier focusing camera. Lead nitrate (cubic, $a = 7.8568 \text{ \AA}$) was used as an internal standard. A scale graduated in 0.1 mm was photographed on the films before their processing. All lanthanoid oxydiacetates except the promethium compound were investigated. The intensities of the powder reflexions were measured for the neodymium compound with a commercial Philips diffractometer using Ni-filtered Cu radiation.

THE REFINEMENT OF THE STRUCTURE

The isostructural lanthanoid oxydiacetate compounds crystallize in the trigonal space group $R32$.² The general positions of this space group are

eighteenfold. The structure contains twelve independent nonhydrogen atoms. Their positional and thermal parameters in NDG and YDG were improved by fullmatrix least-squares refinements. The initial values were taken from Ref. 1. In the function minimized

$$\sum w(|F_o| - |F_c|)^2$$

only reflexions with $0.80 \leq |F_o|/|F_c| \leq 1.25$ were included. The following weighting schemes, chosen according to Hughes,³ were applied in the preliminary refinements

$$1/\sqrt{w} = (A \text{ if } |F_o| \leq A, \text{ else } |F_o|)$$

with $A = 100$ for NDG and 120 for YDG.

The atomic scattering factors used in the calculations were those for C, N, O, Na, and Cl given in *International Tables*.⁴ Those given by Cromer *et al.*⁵ were used for the lanthanoid atoms. The scattering factors of the lanthanoid, chlorine, and sodium atoms were corrected for anomalous dispersion using the corrections $\Delta f'$ and $\Delta f''$ tabulated by Cromer.⁶

The convergence of the refinements was followed by the usual discrepancy indices

$$R = \sum ||F_o| - |F_c|| / \sum |F_o|$$

$$wR = [\sum w(|F_o| - |F_c|)^2 / \sum w|F_o|^2]^{\frac{1}{2}}$$

Reflexions assigned zero weight are included in the calculation of R .

There are two possible absolute configurations of NDG and YDG. They are related, *e.g.*, by a mirror plane at $x - 2y = 0$. With one of them the following three cases were refined for both compounds.

- (a) All atoms have isotropic temperature factors.
- (b) Only the lanthanoid atom has an anisotropic temperature factor.
- (c) All atoms have anisotropic temperature factors.

These refinements converged to the following discrepancy indices for NDG

| | (a) | (b) | (c) |
|------|--------|--------|--------|
| R | 0.0761 | 0.0674 | 0.0587 |
| wR | 0.0682 | 0.0610 | 0.0536 |

It thus seems reasonable to assign anisotropic temperature factors to all nonhydrogen atoms. Since the other absolute configuration of NDG converged to $R(c) = 0.0676$ and $wR(c) = 0.0591$ the first one tried is most probably correct if no systematic errors are present and the weighting scheme is correctly chosen. As mentioned below (p. 3530) it is doubtful if these conditions are obeyed but the first tried configuration of NDG was used for further work.

The weighting scheme was changed to $w = 1/\sigma^2(|F_o|)$ for reflexions obeying the conditions $I > 3\sigma(I)$ and $0.80 \leq |F_o|/|F_c| \leq 1.25$, other reflexions were given zero weights. Three more cycles of least-squares refinement resulted in $R(c) = 0.0613$ and $wR(c) = 0.0450$. The shifts in the parameters were less than 1% of the estimated standard deviations in the last cycle. A three-dimensional difference synthesis showed only a slowly varying background, the highest

peak being about $0.7 \text{ e}/\text{\AA}^3$. It was not possible to locate the hydrogen atoms in the structure.

The same absolute configuration of YDG as the one chosen for NDG converged to the following discrepancy indices in the refinements (a)–(c)

| | (a) | (b) | (c) |
|------|--------|--------|--------|
| R | 0.0697 | 0.0667 | 0.0590 |
| wR | 0.0645 | 0.0623 | 0.0540 |

Anisotropic temperature factors were assigned to all non-hydrogen atoms in YDG too. The other possible configuration converged to $R(a)=0.0842$, $wR(a)=0.0756$ and, consequently, the first one tried was used for further work. Three more cycles of least-squares refinements with the weighting scheme changed as for NDG gave $R(c)=0.0593$, $wR(c)=0.0536$. After the last cycle the shifts in the parameters were less than 1 % of the estimated standard deviations.

Of the 25 reflexions given zero weights in the last cycle of the refinement of YDG only 2 had $I \leq 3\sigma(I)$. Due to the small volume of the crystal of NDG, 87 reflexions had $I \leq 3\sigma(I)$, while a total of 109 were given zero weights. None of the applied weighting schemes seemed to be appropriate; strong reflexions were given too heavy weights. This incorrect weighting might have resulted in somewhat low values of the estimated standard deviations, *e.g.*, in the interatomic distances and angles.

The ultimate positional parameters with estimated standard deviations for all non-hydrogen atoms in NDG and YDG are given in Table 1, and the thermal parameters together with the root-mean-square components along principal axes of the ellipsoids of thermal vibration are given in Table 2. Observed and calculated structure factors are compared in Table 3.

All computations were performed on the CD 3600 computer in Uppsala, Sweden, using the programs PELLE (a program for correction of PAILRED data ⁷), DRF, DATAP2, LALS, DISTAN, ORFFE, ORTEP, and PLANE.⁸

Table 1. Positional parameters with estimated standard deviations in NDG and YDG. The space group is $R32$ (No. 155).³

| Atom | M=Nd | | | M ₂ =Yb | | |
|-------|-------------|------------|-----------|--------------------|------------|-----------|
| | x | y | z | x | y | z |
| M | 0 | 0 | 0 | 0 | 0 | 0 |
| Na(1) | 0.0354 (8) | 0.3687 (8) | 1/6 | 0.0373 (7) | 0.3707 (7) | 1/6 |
| Na(2) | 0 | 0 | 0.2000(2) | 0 | 0 | 0.2018(2) |
| O(1) | -0.2580(10) | 0 | 0 | -0.2511 (9) | 0 | 0 |
| O(2) | -0.0337 (8) | 0.1595 (7) | 0.0595(2) | -0.0234 (7) | 0.1615 (7) | 0.0577(2) |
| O(3) | -0.1706 (8) | 0.2460 (8) | 0.1009(2) | -0.1549 (8) | 0.2542 (9) | 0.0993(2) |
| O(4) | 0.1736 (9) | 0.5961(21) | 0.1131(3) | 0.1723(11) | 0.6009(23) | 0.1127(3) |
| O(5) | 1/3 | 2/3 | 0.0455(4) | 1/3 | 2/3 | 0.0452(4) |
| O(6) | 0.1403 (9) | 0.2179 (9) | 0.1420(2) | 0.1466 (8) | 0.2195(10) | 0.1426(2) |
| C(1) | -0.2946(13) | 0.0926(13) | 0.0324(3) | -0.2818(12) | 0.0959(12) | 0.0316(3) |
| C(2) | -0.1486(24) | 0.1775(24) | 0.0656(2) | -0.1425(17) | 0.1791(17) | 0.0650(3) |
| Cl | 1/3 | 2/3 | 0.0956(1) | 1/3 | 2/3 | 0.0952(1) |

Table 2. Anisotropic temperature factor parameters $\beta_{ij} \times 10^4$ with estimated standard deviations. The expression used is $\exp[-(h^2\beta_{11} + hk\beta_{12} + \dots)]$. Root-mean-square components R_i along principal axes of the ellipsoids of thermal vibration calculated from the values of β_{ij} are also given.

A. NDG

| Atom | β_{11} | β_{22} | β_{33} | β_{12} | β_{13} | β_{23} | $R_1/\text{\AA}$ | $R_2/\text{\AA}$ | $R_3/\text{\AA}$ |
|-------|--------------|--------------|--------------|--------------|--------------|--------------|------------------|------------------|------------------|
| Nd | 50 (1) | 50 (1) | 3(1) | 50 (1) | 0 | 0 | 0.110 | 0.135 | 0.135 |
| Na(1) | 162(10) | 162(10) | 17(1) | 218(22) | 5(3) | -5 (3) | 0.192 | 0.256 | 0.266 |
| Na(2) | 94 (5) | 94 (5) | 7(1) | 94 (5) | 0 | 0 | 0.169 | 0.185 | 0.185 |
| O(1) | 88(12) | 123(27) | 6(1) | 123(27) | -11(4) | -22 (8) | 0.133 | 0.167 | 0.227 |
| O(2) | 88(11) | 95(11) | 6(1) | 90(18) | 6(5) | -13 (5) | 0.136 | 0.180 | 0.211 |
| O(3) | 128(16) | 147(18) | 10(1) | 139(31) | 17(5) | -22 (6) | 0.142 | 0.222 | 0.266 |
| O(4) | 121(12) | 208(28) | 22(1) | 167(48) | 26(7) | 19(16) | 0.194 | 0.272 | 0.312 |
| O(5) | 298(21) | 298(21) | 11(2) | 298(21) | 0 | 0 | 0.212 | 0.329 | 0.329 |
| O(6) | 109(18) | 169(13) | 10(1) | 178(21) | -2(5) | -14 (6) | 0.169 | 0.198 | 0.253 |
| C(1) | 101(18) | 111(18) | 10(1) | 99(31) | -2(8) | -25 (8) | 0.158 | 0.192 | 0.239 |
| C(2) | 99(33) | 28(18) | 7(1) | 40(41) | 9(9) | -7 (8) | 0.094 | 0.158 | 0.215 |
| Cl | 124 (4) | 124 (4) | 8(1) | 124 (4) | 0 | 0 | 0.180 | 0.212 | 0.212 |

B. YDG

| Atom | β_{11} | β_{22} | β_{33} | β_{12} | β_{13} | β_{23} | $R_1/\text{\AA}$ | $R_2/\text{\AA}$ | $R_3/\text{\AA}$ |
|-------|--------------|--------------|--------------|--------------|--------------|--------------|------------------|------------------|------------------|
| Yb | 37 (1) | 37 (1) | 2(0) | 37 (1) | 0 | 0 | 0.089 | 0.114 | 0.114 |
| Na(1) | 123 (8) | 123 (8) | 11(1) | 164(17) | 1(2) | -1 (2) | 0.173 | 0.210 | 0.221 |
| Na(2) | 77 (5) | 77 (5) | 7(1) | 77 (5) | 0 | 0 | 0.165 | 0.167 | 0.167 |
| O(1) | 67 (9) | 61(19) | 3(1) | 61(19) | -5(3) | -10 (6) | 0.096 | 0.155 | 0.158 |
| O(2) | 61 (9) | 74 (9) | 5(1) | 74(15) | -6(4) | 12 (4) | 0.124 | 0.142 | 0.176 |
| O(3) | 94(10) | 110(11) | 7(1) | 119(17) | -4(4) | -26 (4) | 0.121 | 0.179 | 0.224 |
| O(4) | 100(11) | 139(21) | 18(1) | 100(29) | 21(6) | 16(11) | 0.179 | 0.231 | 0.276 |
| O(5) | 289(22) | 289(22) | 5(1) | 289(22) | 0 | 0 | 0.141 | 0.320 | 0.320 |
| O(6) | 99(13) | 140(13) | 6(1) | 141(18) | 3(5) | -8 (5) | 0.142 | 0.182 | 0.225 |
| C(1) | 73(13) | 102(15) | 9(1) | 122(24) | -18(7) | -36 (7) | 0.108 | 0.139 | 0.243 |
| C(2) | 62(22) | 54(20) | 4(1) | 64(40) | 10(6) | -11 (6) | 0.106 | 0.136 | 0.162 |
| Cl | 96 (3) | 96 (3) | 7(1) | 96 (3) | 0 | 0 | 0.167 | 0.185 | 0.185 |

DISCUSSION OF THE STRUCTURE

An outline of the structure was given in Ref. 1. In this section the result of the present investigation and that of Ref. 1 are compared and the differences between NDG and YDG are discussed. Some symmetry-related sites in the structure are designated below by superscripts (i)–(vi) in the following way:

$$\begin{array}{lll}
 \text{(i)} & x-y, \bar{y}, \bar{z} & \text{(ii)} \quad \bar{y}, x-y, z & \text{(iii)} \quad y, x, \bar{z} \\
 \text{(iv)} & y-x, \bar{x}, z & \text{(v)} \quad \bar{x}, y-x, \bar{z} & \text{(vi)} \quad 1-y, 1+x-y, z
 \end{array}$$

where x, y, z are the coordinates of the "crystal-chemical" unit given in Table 1. Selected interatomic distances and angles with estimated standard deviations are given in Table 4.

Table 3. Observed and calculated structure factors for the isostructural compounds NDG and YDG.

| NDG | | | | YDG | | | | NDG | | | | YDG | | | | NDG | | | | YDG | | | | | | | | | |
|-----|---|-----|----------------|----------------|----------------|----------------|---|-----|------|----------------|----------------|----------------|----------------|-----|----|-----|----------------|----------------|----------------|----------------|-----|-----|-----|----------------|----------------|----------------|----------------|-----|----|
| h | k | l | F _o | F _c | F _o | F _c | h | k | l | F _o | F _c | F _o | F _c | h | k | l | F _o | F _c | F _o | F _c | h | k | l | F _o | F _c | F _o | F _c | | |
| 10 | 0 | -14 | 54 | 53 | 87 | 85 | 4 | 0 | 22 | 68 | 66 | 91 | 92 | 9 | -1 | -24 | 56 | 67 | 86 | 84 | 3 | -1 | -20 | 50 | 51 | 58 | 58 | | |
| 10 | 0 | -11 | 36 | 32 | 53 | 51 | 4 | 0 | 25 | 92 | 92 | 112 | 114 | 8 | -1 | -21 | 81 | 77 | 100 | 103 | 3 | -1 | -23 | 113 | 111 | 141 | 144 | | |
| 10 | 0 | -8 | 41 | 37 | 49 | 47 | 4 | 0 | 28 | 77 | 76 | 107 | 111 | 8 | -1 | -18 | 40 | 44 | 64 | 66 | 3 | -1 | -26 | 96 | 99 | 114 | 117 | | |
| 10 | 0 | -6 | 76 | 76 | 91 | 95 | 4 | 0 | 31 | 53 | 49 | 58 | 62 | 8 | -1 | -15 | 64 | 66 | 88 | 88 | 3 | -1 | -29 | 73 | 75 | 99 | 104 | | |
| 10 | 0 | -4 | 79 | 67 | 103 | 104 | 4 | 0 | 34 | 44 | 50 | | | 8 | -1 | -12 | 79 | 75 | 96 | 94 | 3 | -1 | -32 | 50 | 56 | 73 | 75 | | |
| 10 | 0 | 1 | 37 | 73 | 47 | 51 | 3 | 0 | 33 | 62 | 59 | 90 | 80 | 8 | -1 | -9 | 96 | 102 | 130 | 130 | 3 | -1 | -35 | 61 | 60 | 69 | 75 | | |
| 10 | 0 | 4 | 55 | 51 | 79 | 82 | 3 | 0 | 30 | 75 | 71 | 125 | 97 | 8 | -1 | -6 | 63 | 62 | 85 | 82 | 2 | -1 | -36 | 45 | 43 | | | | |
| 10 | 0 | 7 | 52 | 53 | 70 | 73 | 3 | 0 | 27 | 62 | 65 | 83 | 82 | 8 | -1 | -3 | 64 | 65 | 91 | 91 | 2 | -1 | -33 | 68 | 61 | 84 | 82 | | |
| 10 | 0 | 10 | 71 | 65 | 92 | 87 | 3 | 0 | 24 | 133 | 133 | 191 | 169 | 8 | -1 | 0 | 119 | 119 | 148 | 149 | 2 | -1 | -30 | 72 | 74 | 84 | 83 | | |
| 10 | 0 | 13 | 81 | 63 | 81 | 24 | 3 | 0 | 21 | 109 | 110 | 134 | 125 | 8 | -1 | 3 | 77 | 77 | 100 | 99 | 2 | -1 | -27 | 62 | 59 | 89 | 89 | | |
| 9 | 0 | 16 | 58 | 63 | 106 | 108 | 3 | 0 | 18 | 130 | 131 | 169 | 164 | 8 | -1 | 6 | 78 | 79 | 97 | 97 | 2 | -1 | -24 | 64 | 66 | 110 | 116 | | |
| 9 | 0 | 18 | 45 | 43 | 65 | 65 | 3 | 0 | 15 | 14 | 26 | 43 | 42 | 8 | -1 | 9 | 114 | 117 | 144 | 145 | 2 | -1 | -21 | 157 | 155 | 176 | 181 | | |
| 9 | 0 | 15 | 35 | 36 | 62 | 66 | 3 | 0 | 12 | 87 | 99 | 132 | 138 | 8 | -1 | 12 | 71 | 75 | 109 | 108 | 2 | -1 | -18 | 82 | 82 | 111 | 110 | | |
| 9 | 0 | 12 | 61 | 56 | 83 | 83 | 3 | 0 | 9 | 81 | 77 | 112 | 113 | 8 | -1 | 15 | 27 | 37 | 54 | 57 | 2 | -1 | -15 | 126 | 128 | 157 | 164 | | |
| 9 | 0 | 9 | 62 | 58 | 74 | 75 | 3 | 0 | 6 | 356 | 364 | 346 | 369 | 8 | -1 | 18 | 50 | 53 | 72 | 74 | 2 | -1 | -12 | 74 | 71 | 88 | 87 | | |
| 9 | 0 | 6 | 56 | 59 | 77 | 79 | 3 | 0 | 3 | 16 | 21 | 43 | 44 | 8 | -1 | 21 | 86 | 84 | 103 | 104 | 2 | -1 | -9 | 236 | 237 | 245 | 272 | | |
| 9 | 0 | 3 | 58 | 63 | 106 | 108 | 3 | 0 | 0 | 220 | 228 | 253 | 268 | 8 | -1 | 24 | 24 | 24 | 34 | 34 | 2 | -1 | -6 | 63 | 65 | 109 | 108 | | |
| 9 | 0 | 0 | 73 | 67 | 89 | 89 | 3 | 0 | -3 | 240 | 228 | 282 | 251 | 7 | -1 | 29 | 34 | 43 | 49 | 50 | 2 | -1 | -3 | 160 | 157 | 170 | 187 | | |
| 9 | 0 | -3 | 10 | 28 | 34 | 36 | 3 | 0 | -6 | 62 | 63 | 91 | 92 | 7 | -1 | 26 | 64 | 63 | 80 | 83 | 2 | -1 | 0 | 247 | 258 | 255 | 298 | | |
| 9 | 0 | -6 | 87 | 69 | 89 | 90 | 3 | 0 | -9 | 81 | 77 | 112 | 113 | 7 | -1 | 23 | 54 | 55 | 65 | 64 | 11 | -2 | -11 | 48 | 49 | 68 | 62 | | |
| 9 | 0 | -9 | 80 | 87 | 121 | 118 | 3 | 0 | -12 | 209 | 207 | 239 | 235 | 7 | -1 | 20 | 45 | 43 | 65 | 64 | 11 | -2 | -8 | 247 | 257 | 260 | 282 | | |
| 9 | 0 | -12 | 75 | 69 | 96 | 93 | 3 | 0 | -15 | 73 | 67 | 95 | 91 | 7 | -1 | 17 | 54 | 65 | 92 | 95 | 11 | -2 | -5 | 26 | 27 | 44 | 48 | | |
| 9 | 0 | -15 | 36 | 59 | 94 | 99 | 3 | 0 | -18 | 58 | 64 | 76 | 74 | 7 | -1 | 14 | 123 | 128 | 173 | 166 | 11 | -2 | -2 | 45 | 59 | 76 | 79 | | |
| 9 | 0 | -18 | 21 | 27 | 55 | 56 | 3 | 0 | -21 | 114 | 113 | 134 | 131 | 7 | -1 | 11 | 78 | 87 | 93 | 92 | 10 | -2 | -1 | 47 | 46 | 62 | 68 | | |
| 9 | 0 | -21 | 54 | 47 | 53 | 57 | 3 | 0 | -24 | 144 | 145 | 179 | 181 | 7 | -1 | 8 | 24 | 27 | 36 | 38 | 10 | -2 | 1 | 54 | 52 | 70 | 73 | | |
| 8 | 0 | -25 | 63 | 74 | 81 | 81 | 3 | 0 | -27 | 22 | 29 | 36 | 39 | 7 | -1 | 5 | 113 | 114 | 149 | 149 | 10 | -2 | 4 | 44 | 41 | 54 | 53 | | |
| 8 | 0 | -22 | 38 | 36 | 67 | 68 | 3 | 0 | -30 | 115 | 111 | 145 | 140 | 7 | -1 | 2 | 101 | 105 | 140 | 136 | 10 | -2 | 1 | 53 | 53 | 73 | 73 | | |
| 8 | 0 | -19 | 58 | 50 | 81 | 81 | 3 | 0 | -33 | 46 | 55 | 69 | 70 | 7 | -1 | -1 | 47 | 50 | 72 | 72 | 11 | -2 | 10 | 57 | 53 | 79 | 83 | | |
| 8 | 0 | -16 | 72 | 68 | 80 | 86 | 2 | 0 | -34 | 28 | 30 | 38 | 39 | 7 | -1 | -4 | 114 | 114 | 141 | 141 | 10 | -2 | 13 | 25 | 38 | 56 | 61 | | |
| 8 | 0 | -13 | 75 | 75 | 84 | 85 | 2 | 0 | -37 | 51 | 54 | 75 | 69 | 7 | -1 | -7 | 75 | 67 | 98 | 98 | 9 | -2 | 18 | 49 | 48 | 69 | 70 | | |
| 8 | 0 | -10 | 123 | 124 | 154 | 155 | 2 | 0 | -40 | 89 | 93 | 165 | 121 | 7 | -1 | -10 | 85 | 80 | 104 | 97 | 10 | -2 | 15 | 59 | 58 | 78 | 71 | | |
| 8 | 0 | -7 | 82 | 83 | 104 | 105 | 2 | 0 | -43 | 128 | 132 | 150 | 149 | 7 | -1 | -13 | 36 | 44 | 60 | 59 | 10 | -2 | 12 | 63 | 58 | 84 | 75 | | |
| 8 | 0 | -4 | 85 | 85 | 102 | 102 | 2 | 0 | -46 | 63 | 68 | 91 | 83 | 7 | -1 | -16 | 114 | 110 | 150 | 146 | 9 | -2 | 9 | 54 | 53 | 74 | 73 | | |
| 8 | 0 | -1 | 43 | 42 | 81 | 85 | 2 | 0 | -49 | 87 | 87 | 133 | 123 | 7 | -1 | -19 | 94 | 92 | 123 | 123 | 10 | -2 | 6 | 37 | 44 | 70 | 65 | | |
| 8 | 0 | 2 | 73 | 70 | 106 | 106 | 2 | 0 | -52 | 120 | 119 | 200 | 226 | 7 | -1 | -22 | 16 | 36 | 46 | 46 | 10 | -2 | 3 | 59 | 62 | 78 | 75 | | |
| 8 | 0 | 5 | 119 | 119 | 149 | 149 | 2 | 0 | -55 | 181 | 181 | 243 | 243 | 7 | -1 | -25 | 38 | 43 | 63 | 63 | 9 | -2 | 1 | 62 | 63 | 77 | 79 | | |
| 8 | 0 | 8 | 68 | 69 | 89 | 89 | 2 | 0 | -58 | 101 | 104 | 215 | 213 | 7 | -1 | -28 | 61 | 55 | 68 | 73 | 10 | -2 | -3 | 48 | 45 | 60 | 64 | | |
| 8 | 0 | 11 | 45 | 46 | 80 | 82 | 2 | 0 | -61 | 199 | 211 | 232 | 236 | 7 | -1 | -31 | 45 | 44 | 63 | 65 | 10 | -2 | -6 | 36 | 33 | 58 | 57 | | |
| 8 | 0 | 14 | 68 | 68 | 83 | 83 | 2 | 0 | -64 | 63 | 68 | 91 | 83 | 7 | -1 | -34 | 70 | 71 | 87 | 88 | 9 | -2 | 13 | 63 | 63 | 80 | 80 | | |
| 8 | 0 | 17 | 41 | 59 | 60 | 64 | 2 | 0 | -67 | 19 | 23 | 33 | 32 | 6 | -1 | -37 | 67 | 70 | 95 | 94 | 10 | -2 | -12 | 60 | 51 | 75 | 77 | | |
| 8 | 0 | 20 | 71 | 72 | 83 | 87 | 2 | 0 | -70 | 210 | 211 | | | 6 | -1 | -40 | 48 | 56 | 78 | 77 | 10 | -2 | -15 | 56 | 62 | 73 | 78 | | |
| 8 | 0 | 23 | 73 | 79 | 95 | 95 | 2 | 0 | -73 | 459 | 461 | 401 | 486 | 6 | -1 | -43 | 80 | 106 | 107 | 101 | 10 | -2 | -18 | 43 | 46 | 61 | 62 | | |
| 7 | 0 | 28 | 61 | 60 | | | 2 | 0 | -76 | 19 | 17 | 22 | 22 | 6 | -1 | -46 | 124 | 126 | 157 | 154 | 9 | -2 | -22 | 41 | 39 | 58 | 58 | | |
| 7 | 0 | 25 | 57 | 51 | 66 | 61 | 2 | 0 | -79 | 165 | 172 | 199 | 209 | 6 | -1 | -49 | 88 | 86 | 113 | 110 | 9 | -2 | -25 | 37 | 30 | 73 | 76 | | |
| 7 | 0 | 22 | 48 | 44 | | | 2 | 0 | -82 | 142 | 145 | 187 | 187 | 6 | -1 | -52 | 15 | 18 | 23 | 23 | 8 | -2 | -28 | 58 | 65 | 77 | 79 | | |
| 7 | 0 | 19 | 77 | 74 | 111 | 116 | 2 | 0 | -85 | 77 | 84 | 107 | 107 | 6 | -1 | -55 | 209 | 205 | 243 | 234 | 9 | -2 | -31 | 33 | 35 | 62 | 62 | | |
| 7 | 0 | 16 | 101 | 102 | 131 | 129 | 2 | 0 | -88 | 20 | 110 | 108 | 121 | 121 | 6 | -1 | -58 | 89 | 89 | 118 | 118 | 9 | -2 | -34 | 58 | 62 | 85 | 87 | |
| 7 | 0 | 13 | 119 | 116 | 149 | 149 | 2 | 0 | -91 | 42 | 49 | 62 | 62 | 6 | -1 | -61 | 111 | 111 | 140 | 139 | 9 | -2 | -37 | 44 | 44 | 60 | 61 | | |
| 7 | 0 | 10 | 135 | 132 | 161 | 159 | 2 | 0 | -94 | 26 | 85 | 84 | 106 | 105 | 6 | -1 | -64 | 89 | 90 | 126 | 126 | 9 | -2 | -40 | 83 | 82 | 101 | 103 | |
| 7 | 0 | 7 | 59 | 67 | 88 | 89 | 2 | 0 | -97 | 19 | 23 | 37 | 37 | 6 | -1 | -67 | 81 | 79 | 113 | 115 | 9 | -2 | -43 | 67 | 68 | 74 | 78 | | |
| 7 | 0 | 4 | 108 | 113 | 148 | 148 | 2 | 0 | -100 | 116 | 121 | 163 | 163 | 6 | -1 | -70 | 60 | 63 | 84 | 84 | 10 | -2 | -46 | 82 | 84 | 108 | 108 | | |
| 7 | 0 | 1 | 82 | 80 | 105 | 106 | 2 | 0 | -103 | 35 | 95 | 90 | 123 | 112 | 6 | -1 | -73 | 84 | 85 | 101 | 100 | 9 | -2 | -49 | 114 | 107 | 130 | 133 | |
| 7 | 0 | -2 | 114 | 110 | 161 | 163 | 1 | 0 | -106 | 34 | 82 | 77 | 112 | 108 | 6 | -1 | -76 | 116 | 122 | 128 | 141 | 144 | 9 | -2 | -52 | 40 | 38 | 67 | 75 |
| 7 | 0 | -5 | 88 | 84 | | | 1 | 0 | -109 | 23 | 74 | 98 | 84 | 6 | -1 | -79 | 103 | 108 | 148 | 149 | 9 | -2 | -55 | 58 | 53 | 75 | 79 | | |
| 7 | 0 | -8 | 66 | 66 | 90 | 92 | 1 | 0 | -112 | 28 | 70 | 74 | 94 | 93 | 6 | -1 | -82 | 84 | 28 | 45 | 45 | 9 | -2 | -58 | 94 | 94 | 114 | 118 | |
| 7 | 0 | -11 | 87 | 92 | 109 | 108 | 1 | 0 | -115 | 25 | 58 | 56 | 78 | 74 | 6 | -1 | -85 | 25 | 60 | 58 | 71 | 73 | 9 | -2 | -61 | 40 | 39 | 54 | 54 |
| 7 | 0 | -14 | 120 | 116 | 149 | 145 | 1 | 0 | -118 | 62 | 62 | 94 | 90 | 6 | -1 | -88 | 62 | 68 | 90 | 78 | 9 | -2 | -64 | 62 | 63 | 84 | 85 | | |
| 7 | 0 | -17 | 49 | 57 | 80 | 80 | 1 | 0 | -121 | 19 | 131 | 131 | 173 | 173 | 6 | -1 | -91 | 31 | 68 | 62 | 79 | 80 | 8 | -2 | 25 | 64 | 68 | 88 | 84 |
| 7 | 0 | -20 | 53 | 63 | 99 | 96 | 1 | 0 | -124 | 16 | 239 | 237 | 266 | 269 | 6 | -1 | -94 | 33 | 62 | 63 | 94 | 89 | 8 | -2 | 25 | 64 | 68 | 88 | 84 |
| 7 | 0 | -23 | 83 | 87 | 116 | 116 | 1 | 0 | -127 | 17 | 163 | 190 | 192 | 6 | -1 | -97 | 30 | 76 | 95 | 80 | | | | | | | | | |

Table 3. Continued.

| NDG | | | | | YDG | | | | | NDG | | | | | YDG | | | | | NDG | | | | | YDG | | | | |
|-----|----|-----|----------------|----------------|----------------|----------------|----------------|----------------|-----|-----|-----|----------------|----------------|----------------|----------------|-----|-----|-----|----------------|----------------|----------------|----------------|-----|-----|-----|----------------|----------------|----------------|----------------|
| h | k | l | F _o | F _c | F _o | F _c | F _o | F _c | h | k | l | F _o | F _c | F _o | F _c | h | k | l | F _o | F _c | F _o | F _c | h | k | l | F _o | F _c | F _o | F _c |
| 5 | -2 | -23 | 76 | 72 | 96 | 96 | 10 | -3 | -2 | 77 | 76 | 93 | 93 | 7 | -3 | 4 | 102 | 99 | 135 | 127 | 9 | -4 | -17 | 27 | 41 | 63 | 63 | | |
| 5 | -2 | -20 | 35 | 34 | 50 | 49 | 10 | -3 | -5 | 76 | 77 | 101 | 101 | 7 | -3 | 7 | 119 | 122 | 159 | 155 | 9 | -4 | -14 | 59 | 70 | 97 | 98 | | |
| 5 | -2 | -17 | 69 | 72 | 95 | 94 | 10 | -3 | -8 | 49 | 54 | 82 | 85 | 7 | -3 | 10 | 116 | 114 | 152 | 150 | 9 | -4 | -11 | 88 | 88 | 119 | 113 | | |
| 5 | -2 | -14 | 159 | 163 | 156 | 150 | 10 | -3 | -11 | 42 | 58 | 56 | 7 | -3 | 13 | 53 | 54 | 58 | 61 | 9 | -4 | -8 | 0 | 21 | 44 | 42 | | | |
| 5 | -2 | -11 | 87 | 87 | 113 | 115 | 10 | -3 | -14 | 88 | 79 | 101 | 105 | 7 | -3 | 16 | 142 | 141 | 154 | 165 | 9 | -4 | -5 | 127 | 116 | 166 | 165 | | |
| 5 | -2 | -8 | 92 | 90 | 117 | 115 | 10 | -3 | -17 | 29 | 38 | 39 | 45 | 7 | -3 | 19 | 85 | 89 | 110 | 117 | 9 | -4 | -2 | 97 | 96 | 134 | 132 | | |
| 5 | -2 | -5 | 222 | 221 | 236 | 261 | 10 | -3 | -20 | 45 | 47 | 58 | 62 | 7 | -3 | 22 | 50 | 47 | 59 | 66 | 9 | -4 | 1 | 156 | 152 | 180 | 85 | | |
| 5 | -2 | -2 | 62 | 68 | 70 | 71 | 9 | -3 | -24 | 47 | 53 | 60 | 69 | 7 | -3 | 25 | 56 | 59 | 62 | 76 | 9 | -4 | 4 | 76 | 78 | 97 | 99 | | |
| 5 | -2 | 1 | 142 | 136 | 156 | 156 | 9 | -3 | -21 | 67 | 70 | 87 | 95 | 7 | -3 | 28 | 74 | 73 | 69 | 102 | 9 | -4 | 7 | 84 | 91 | 115 | 116 | | |
| 5 | -2 | 4 | 101 | 96 | 115 | 121 | 9 | -3 | -18 | 56 | 54 | 74 | 73 | 6 | -3 | 30 | 76 | 72 | 19 | 87 | 9 | -4 | 10 | 69 | 73 | 94 | 95 | | |
| 5 | -2 | 7 | 97 | 97 | 124 | 136 | 9 | -3 | -15 | 36 | 42 | 48 | 48 | 6 | -3 | 27 | 23 | 36 | 25 | 53 | 9 | -4 | 13 | 37 | 47 | 69 | 70 | | |
| 5 | -2 | 10 | 72 | 74 | 94 | 104 | 9 | -3 | -12 | 45 | 50 | 74 | 75 | 6 | -3 | 24 | 63 | 66 | 76 | 98 | 9 | -4 | 16 | 73 | 73 | 97 | 102 | | |
| 5 | -2 | 13 | 90 | 92 | 112 | 121 | 9 | -3 | -9 | 97 | 98 | 136 | 134 | 6 | -3 | 21 | 121 | 125 | 124 | 137 | 9 | -4 | 19 | 67 | 64 | 86 | 90 | | |
| 5 | -2 | 16 | 104 | 108 | 128 | 134 | 9 | -3 | -6 | 70 | 72 | 94 | 95 | 6 | -3 | 18 | 59 | 60 | 81 | 79 | 9 | -4 | 22 | 39 | 29 | 34 | 41 | | |
| 5 | -2 | 19 | 116 | 115 | 139 | 147 | 9 | -3 | -3 | 56 | 56 | 78 | 74 | 6 | -3 | 15 | 63 | 56 | 78 | 78 | 9 | -4 | 25 | 55 | 61 | 81 | 85 | | |
| 5 | -2 | 22 | 49 | 54 | 60 | 63 | 9 | -3 | 0 | 104 | 105 | 146 | 145 | 6 | -3 | 12 | 96 | 99 | 135 | 123 | 9 | -4 | 28 | 51 | 53 | 73 | 73 | | |
| 5 | -2 | 25 | 75 | 78 | 101 | 99 | 9 | -3 | 3 | 0 | 21 | 41 | 19 | 6 | -3 | 9 | 121 | 123 | 167 | 151 | 8 | -4 | 31 | 59 | 64 | 89 | 93 | | |
| 5 | -2 | 28 | 64 | 66 | 92 | 95 | 9 | -3 | 6 | 30 | 37 | 42 | 43 | 6 | -3 | 6 | 98 | 100 | 148 | 134 | 8 | -4 | 34 | 72 | 70 | 88 | 91 | | |
| 5 | -2 | 31 | 52 | 53 | 64 | 66 | 9 | -3 | 9 | 89 | 90 | 125 | 123 | 6 | -3 | 3 | 17 | 19 | 30 | 26 | 8 | -4 | 37 | 62 | 69 | 103 | 100 | | |
| 5 | -2 | 34 | 39 | 53 | 70 | 75 | 9 | -3 | 12 | 67 | 77 | 100 | 107 | 6 | -3 | 0 | 241 | 246 | 243 | 251 | 8 | -4 | 40 | 100 | 96 | 126 | 127 | | |
| 5 | -2 | 37 | 52 | 52 | 71 | 73 | 9 | -3 | 15 | 32 | 41 | 53 | 58 | 12 | -4 | 7 | 51 | 51 | 70 | 73 | 8 | -4 | 43 | 78 | 109 | 111 | | | |
| 5 | -2 | 40 | 65 | 69 | 93 | 92 | 9 | -3 | 18 | 51 | 49 | 56 | 62 | 12 | -4 | 4 | 13 | 41 | 54 | 59 | 8 | -4 | 46 | 108 | 110 | 152 | 150 | | |
| 4 | -2 | 24 | 65 | 69 | 93 | 92 | 9 | -3 | 21 | 62 | 64 | 76 | 67 | 12 | -4 | 1 | 47 | 50 | 73 | 72 | 8 | -4 | 49 | 69 | 73 | 99 | 97 | | |
| 4 | -2 | 21 | 122 | 121 | 141 | 140 | 9 | -3 | 24 | 53 | 50 | 60 | 67 | 12 | -4 | -2 | 43 | 49 | 72 | 75 | 8 | -4 | 52 | 73 | 83 | 109 | 106 | | |
| 4 | -2 | 18 | 52 | 57 | 73 | 71 | 8 | -3 | 26 | 72 | 71 | 80 | 88 | 12 | -4 | -5 | 80 | 78 | 110 | 107 | 8 | -4 | 55 | 175 | 196 | 196 | | | |
| 4 | -2 | 15 | 109 | 114 | 139 | 135 | 8 | -3 | 29 | 65 | 62 | 82 | 84 | 12 | -4 | -8 | 30 | 36 | 60 | 57 | 12 | -5 | 11 | 54 | 49 | 46 | 47 | | |
| 4 | -2 | 12 | 103 | 100 | 137 | 134 | 8 | -3 | 32 | 55 | 49 | 65 | 65 | 11 | -4 | -15 | 34 | 46 | 88 | 66 | 12 | -5 | 8 | 35 | 46 | 47 | | | |
| 4 | -2 | 9 | 144 | 150 | 189 | 190 | 8 | -3 | 35 | 40 | 40 | 55 | 64 | 11 | -4 | -12 | 51 | 57 | 82 | 80 | 12 | -5 | 5 | 45 | 51 | 72 | 74 | | |
| 4 | -2 | 6 | 121 | 114 | 149 | 133 | 8 | -3 | 38 | 14 | 119 | 119 | 149 | 147 | 11 | -4 | -9 | 61 | 66 | 81 | 79 | 12 | -5 | 2 | 53 | 53 | 76 | 79 | |
| 4 | -2 | 3 | 154 | 158 | 202 | 189 | 8 | -3 | 41 | 11 | 125 | 121 | 147 | 148 | 11 | -4 | -6 | 58 | 58 | 71 | 72 | 12 | -5 | -1 | 33 | 56 | 83 | 82 | |
| 4 | -2 | 0 | 205 | 208 | 264 | 238 | 8 | -3 | 44 | 8 | 137 | 20 | 54 | 49 | 11 | -4 | -3 | 44 | 44 | 54 | 57 | 12 | -5 | -4 | 36 | 47 | 65 | 68 | |
| 12 | -3 | 6 | 41 | 50 | 64 | 43 | 8 | -3 | 5 | 108 | 112 | 146 | 140 | 11 | -4 | 0 | 92 | 90 | 116 | 117 | 12 | -5 | -7 | 49 | 51 | 71 | 69 | | |
| 12 | -3 | 3 | 27 | 30 | 43 | 41 | 8 | -3 | 2 | 86 | 83 | 123 | 113 | 11 | -4 | 3 | 47 | 51 | 69 | 71 | 12 | -5 | -10 | 54 | 51 | 61 | 68 | | |
| 12 | -3 | 0 | 67 | 62 | 79 | 81 | 8 | -3 | -1 | 58 | 61 | 94 | 87 | 11 | -4 | -5 | 69 | 57 | 75 | 75 | 11 | -5 | -17 | 59 | 44 | 63 | 62 | | |
| 12 | -3 | -3 | 49 | 51 | 72 | 74 | 8 | -3 | -4 | 102 | 105 | 142 | 135 | 11 | -4 | 9 | 62 | 59 | 78 | 80 | 11 | -5 | -14 | 53 | 63 | 91 | 87 | | |
| 12 | -3 | -6 | 32 | 40 | 46 | 48 | 8 | -3 | -7 | 124 | 125 | 157 | 146 | 11 | -4 | 12 | 63 | 56 | 72 | 75 | 11 | -5 | -11 | 50 | 52 | 73 | 70 | | |
| 11 | -3 | -16 | 65 | 60 | 86 | 86 | 8 | -3 | -10 | 75 | 75 | 110 | 105 | 11 | -4 | -13 | 69 | 57 | 74 | 74 | 11 | -5 | -8 | 19 | 33 | 41 | 43 | | |
| 11 | -3 | -13 | 25 | 37 | 48 | 50 | 8 | -3 | -13 | 52 | 48 | 77 | 73 | 10 | -4 | 20 | 41 | 53 | 80 | 72 | 11 | -5 | -5 | 66 | 74 | 94 | 93 | | |
| 11 | -3 | -10 | 49 | 56 | 74 | 77 | 8 | -3 | -16 | 100 | 98 | 118 | 120 | 10 | -4 | 17 | 46 | 44 | 65 | 62 | 11 | -5 | -2 | 30 | 50 | 73 | 71 | | |
| 11 | -3 | -7 | 61 | 61 | 86 | 91 | 8 | -3 | -19 | 90 | 92 | 116 | 121 | 10 | -4 | 14 | 66 | 68 | 104 | 98 | 11 | -5 | 1 | 47 | 51 | 65 | 66 | | |
| 11 | -3 | -4 | 55 | 58 | 69 | 74 | 8 | -3 | -22 | 43 | 41 | 46 | 60 | 10 | -4 | 11 | 60 | 59 | 77 | 77 | 11 | -5 | 4 | 67 | 65 | 85 | 85 | | |
| 11 | -3 | -1 | 63 | 66 | 76 | 79 | 8 | -3 | -25 | 76 | 66 | 61 | 86 | 10 | -4 | 8 | 64 | 59 | 77 | 78 | 11 | -5 | 7 | 63 | 66 | 91 | 89 | | |
| 11 | -3 | 2 | 67 | 64 | 84 | 87 | 8 | -3 | -28 | 46 | 45 | 46 | 67 | 10 | -4 | 5 | 89 | 86 | 108 | 104 | 11 | -5 | 10 | 44 | 55 | 84 | 78 | | |
| 11 | -3 | 5 | 60 | 62 | 86 | 90 | 7 | -3 | -29 | 44 | 46 | 27 | 61 | 10 | -4 | 2 | 91 | 83 | 89 | 91 | 11 | -5 | 13 | 48 | 35 | 53 | 50 | | |
| 11 | -3 | 8 | 29 | 34 | 51 | 51 | 7 | -3 | -32 | 78 | 78 | 63 | 105 | 10 | -4 | -1 | 24 | 31 | 51 | 51 | 11 | -5 | 16 | 52 | 60 | 89 | 84 | | |
| 11 | -3 | 11 | 49 | 48 | 60 | 64 | 7 | -3 | -35 | 69 | 72 | 78 | 100 | 10 | -4 | 4 | 53 | 58 | 85 | 84 | 10 | -5 | 21 | 50 | 66 | 87 | 60 | | |
| 11 | -3 | 14 | 66 | 70 | 92 | 100 | 7 | -3 | -38 | 96 | 95 | 109 | 113 | 10 | -4 | -7 | 62 | 60 | 80 | 78 | 10 | -5 | 9 | 66 | 89 | 88 | 45 | 60 | |
| 11 | -3 | 17 | 47 | 51 | 61 | 64 | 7 | -3 | -41 | 46 | 39 | 46 | 45 | 10 | -4 | -10 | 86 | 79 | 102 | 99 | 10 | -5 | 15 | 35 | 47 | 66 | 69 | | |
| 11 | -3 | 20 | 73 | 73 | 91 | 92 | 7 | -3 | -44 | 150 | 141 | 162 | 162 | 10 | -4 | -13 | 43 | 37 | 55 | 54 | 10 | -5 | 12 | 37 | 47 | 61 | 65 | | |
| 11 | -3 | 23 | 30 | 37 | 53 | 53 | 7 | -3 | -47 | 132 | 97 | 131 | 130 | 10 | -4 | -16 | 62 | 60 | 80 | 78 | 10 | -5 | 9 | 66 | 89 | 111 | 113 | | |
| 11 | -3 | 26 | 57 | 57 | 82 | 81 | 7 | -3 | -50 | 67 | 68 | 99 | 90 | 10 | -4 | -19 | 60 | 60 | 86 | 83 | 10 | -5 | 6 | 40 | 49 | 60 | 62 | | |
| 11 | -3 | 29 | 43 | 51 | 73 | 76 | 7 | -3 | -53 | 134 | 127 | 179 | 160 | 10 | -4 | -22 | 56 | 49 | 66 | 87 | 10 | -5 | 3 | 39 | 51 | 76 | 79 | | |
| 11 | -3 | 4 | 76 | 77 | 95 | 96 | 7 | -3 | -56 | 135 | 136 | 184 | 167 | 9 | -4 | -23 | 62 | 65 | 86 | 87 | 10 | -5 | 0 | 80 | 75 | 104 | 104 | | |
| 11 | -3 | 1 | 39 | 42 | 56 | 57 | 7 | -3 | -59 | 32 | 36 | 44 | 38 | 9 | -4 | -20 | 42 | 49 | 63 | 66 | 10 | -5 | 0 | 80 | 75 | 104 | 104 | | |

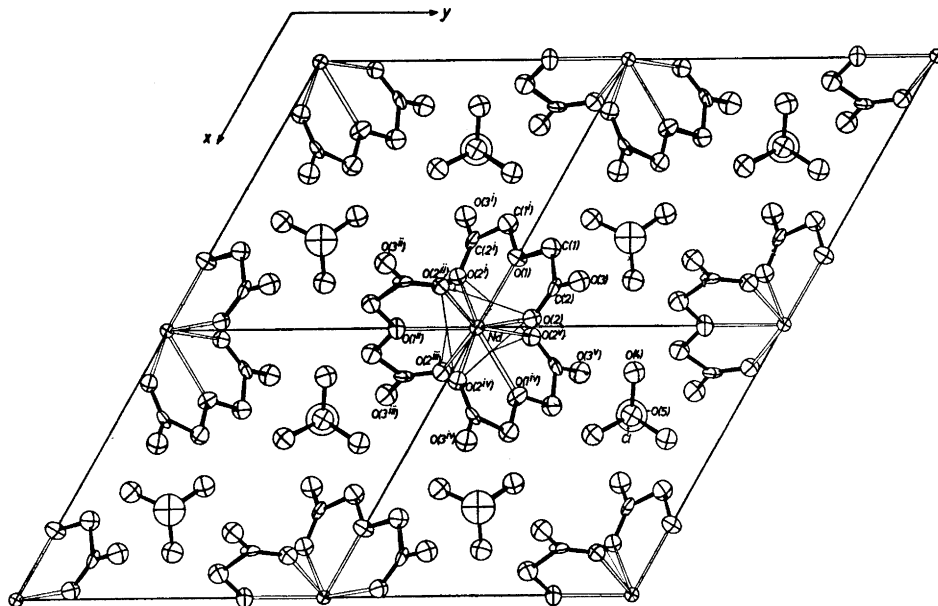


Fig. 1. A projection of NDG on (001) showing the layer around $z=0$ containing the tris(oxodiacetato) complexes and the perchlorate ions. The figure is drawn by the program ORTEP³ representing the atoms by "thermal ellipsoids" scaled to include 50% of the probability distribution.

Table 4. Selected interatomic distances (Å) and angles (°) with estimated standard deviations in NDG and YDG.

A. The coordination polyhedron around the lanthanoid ion.

| Distance | Nd | Yb | Distance | Nd | Yb |
|-----------|-----------|----------|--|-----------|-----------|
| M—O(1) | 2.523(10) | 2.431(9) | O(1)—O(2 ⁱⁱ) | 3.026 (9) | 2.851 (8) |
| M—O(2) | 2.428 (6) | 2.339(6) | O(2)—O(2 ⁱⁱ) | 3.026(11) | 2.924(11) |
| O(1)—O(2) | 2.582 (8) | 2.546(8) | O(2 ⁱ)—O(2 ⁱⁱ) | 3.419(11) | 3.262(11) |

B. The ligand.

| Distance | Nd | Yb | Angle | Nd | Yb |
|-----------|-----------|-----------|------------------------------|------------|------------|
| O(1)—C(1) | 1.454(12) | 1.419(11) | C(1)—O(1)—C(1 ⁱ) | 112.2(1.0) | 115.1(1.0) |
| C(1)—C(2) | 1.557(20) | 1.503(16) | O(1)—C(1)—C(2) | 105.5(1.0) | 108.3(0.9) |
| C(2)—O(2) | 1.234(22) | 1.264(16) | C(1)—C(2)—O(2) | 119.8(1.2) | 117.5(0.9) |
| C(2)—O(3) | 1.281(16) | 1.249(13) | C(1)—C(2)—O(3) | 113.7(1.5) | 118.1(1.2) |
| | | | O(2)—C(2)—O(3) | 125.9(1.3) | 124.3(1.0) |

C. The sodium coordination.

| Distance | Nd | Yb | Distance | Nd | Yb |
|------------|-----------|-----------|-------------|----------|----------|
| Na(1)—O(3) | 2.559 (8) | 2.492 (7) | Na(1)—Na(2) | 3.572(3) | 3.562(3) |
| Na(1)—O(4) | 2.463(15) | 2.461(16) | Na(2)—O(3) | 2.337(7) | 2.390(7) |
| Na(1)—O(6) | 2.287(10) | 2.294(10) | Na(2)—O(6) | 2.489(8) | 2.504(8) |

D. The perchlorate ion.

| Distance | Nd | Yb | Angle | Nd | Yb |
|----------|-----------|-----------|-----------------------------|------------|------------|
| Cl—O(4) | 1.444 (8) | 1.444 (9) | O(4)—Cl—O(4 ^{vi}) | 108.8(0.3) | 109.1(0.4) |
| Cl—O(5) | 1.418(12) | 1.404(12) | O(4)—Cl—O(5) | 110.1(0.3) | 109.9(0.4) |

E. Possible hydrogen bonds.

| Distance | Nd | Yb | Distance | Nd | Yb |
|--------------------------------|------------|------------|--------------------------------|------------|------------|
| O(6)—O(2) | 2.777 (9) | 2.789 (8) | O(6)—O(4 ^{vi}) | 3.259(19) | 3.157(20) |
| O(6)—O(3 ^{iv}) | 3.180(10) | 3.150(10) | O(6)—O(2 ^{iv}) | 3.147 (9) | 3.193 (9) |
| Angle | | | Angle | | |
| O(2)—O(6)—O(3 ^{iv}) | 98.0(0.3) | 94.8(0.3) | Na(1)—O(6)—O(4 ^{vi}) | 75.5(0.3) | 75.2(0.3) |
| O(2)—O(6)—O(4 ^{vi}) | 94.8(0.3) | 92.9(0.3) | Na(2)—O(6)—O(2) | 111.1(0.3) | 111.6(0.3) |
| O(2)—O(6)—O(2 ^{iv}) | 61.1(0.3) | 58.2(0.3) | Na(2)—O(6)—O(3 ^{iv}) | 105.2(0.3) | 105.1(0.3) |
| Na(1)—O(6)—O(2) | 87.9(0.3) | 88.0(0.3) | Na(2)—O(6)—O(4 ^{vi}) | 152.9(0.3) | 153.8(0.3) |
| Na(1)—O(6)—O(3 ^{iv}) | 153.3(0.4) | 156.3(0.4) | | | |

The structure is composed of layers, perpendicular to the c axis, containing the mononuclear tris(oxydiacetato)lanthanoidate complexes and the perchlorate ions, Fig. 1, alternating with layers containing the sodium ions and water molecules, Fig. 2. As is seen in Table 4. A, there is a significant contrac-

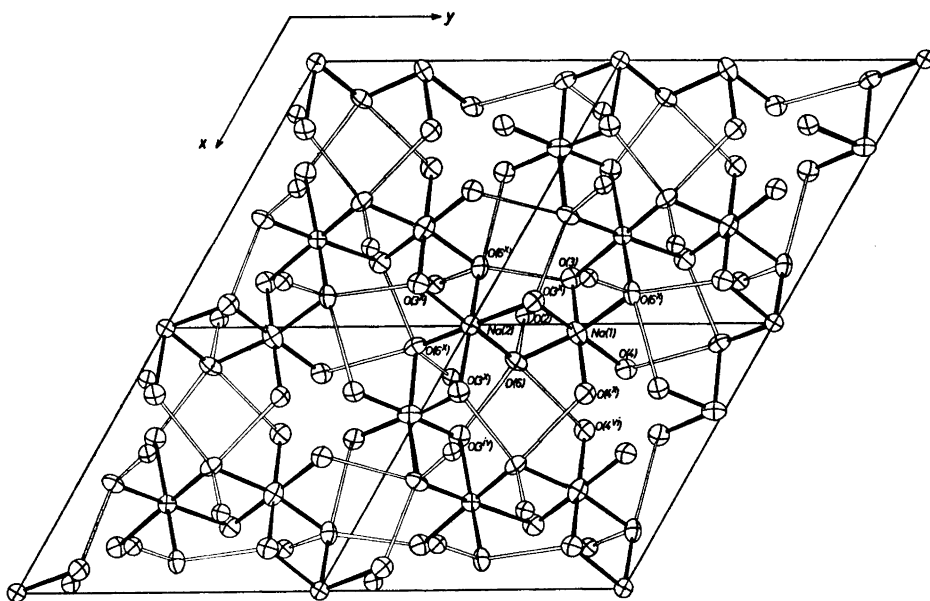


Fig. 2. A projection on (001) of the layer around $z=1/6$ in NDG showing the coordination around the sodium ions. Possible hydrogen bond distances, including $O(6) - O(4^{vi})$, are represented by unfilled sticks. Some symmetry-related sites not given in the text are designated by the superscript (x). The figure is drawn as Fig. 1.

tion of the coordination polyhedron around the nine-coordinated lanthanoid ion between NDG and YDG. The smallest decrease is in the distance $O(1) - O(2)$ where the atoms belong to the same ligand. The differences in both the $M - O(1)$ and the $M - O(2)$ bond distances between NDG and YDG are appreciably less than might be expected from the lanthanoid contraction (see below p. 3540). The triangular faces of the trigonal prism are rotated $18.8 \pm 0.3^\circ$ relative to each other in NDG but this angle has decreased to $13.3 \pm 0.3^\circ$ in YDG. On the other hand the angle between $M - O(2)$ and the c axis is only changed from $46.0 \pm 0.1^\circ$ to $46.2 \pm 0.1^\circ$. The distance between the triangular faces of the prism decreases from $3.371 \pm 0.011 \text{ \AA}$ in NDG to $3.238 \pm 0.011 \text{ \AA}$ in YDG.

The ligand is not much influenced by the change Nd–Yb (Table 4. B, Fig. 1). The deviations from the least-squares planes through the non-hydrogen atoms of the ligand in the two compounds are given in Table 5. The atoms are nearly coplanar. The distances and angles obtained for the ligand in the gadolinium compound were judged as the “best” set in Ref. 1. When comparing the sets obtained from NDG and YDG with each other, with the gadolinium

Table 5. The deviations (in Å) from the least-squares planes through the non-hydrogen atoms of the ligands in NDG and YDG. The lower sign refer to the superscripted atoms. O(1) and M are situated on the same twofold axis.

| Atom | Nd | Yb | Atom | Nd | Yb |
|--------------------------|--------|--------|--------------------------|--------|--------|
| O(1) | 0.000 | 0.000 | O(2), O(2 ⁱ) | ±0.044 | ±0.046 |
| C(1), C(1 ⁱ) | ±0.063 | ±0.074 | O(3), O(3 ⁱ) | ∓0.089 | ∓0.077 |
| C(2), C(2 ⁱ) | ±0.061 | ±0.035 | M | 0.000 | 0.000 |

set, and with the expected distances and angles¹ one may conclude that the ligand is slightly more distorted in NDG than in YDG.

The coordination around the sodium ions is shown in Fig. 2. The sodium-oxygen bond distances (Table 4.C) are almost the same as those given in Ref. 1. In both compounds the O–Na–O bond angles with adjacent oxygen atoms are in the range 81–107° and the twelve different oxygen–oxygen “contact” distances along the edges of the octahedra around Na(1) and Na(2) are in the range 3.24–3.99 Å.

The oxygen–chlorine bond lengths in the perchlorate ion (Table 4.D) are somewhat shorter than those given in the literature.⁹ However, they are not corrected for thermal motion. Assuming riding motion (oxygen on chlorine) the following values were obtained

| | NDG | YDG |
|---------|---------|---------|
| Cl–O(4) | 1.475 Å | 1.468 Å |
| Cl–O(5) | 1.462 Å | 1.453 Å |

These distances are in good agreement with those given in Ref. 9, and the differences between Cl–O(4) and Cl–O(5) are less than those in Table 4.D. The bond distances and bond angles are compatible with a tetrahedral perchlorate ion in both compounds.

In Table 4.E all distances less than 3.20 Å in YDG and NDG between the water oxygen atom O(6) and other oxygen atoms are listed. O(6)–O(2) is the only short distance and it has almost the same length in both compounds. Assuming a located, linear hydrogen bond between O(6) and O(2), the second hydrogen atom of O(6) may interact with O(3^{iv}) and O(4^{vi}) in YDG. In NDG the distance O(6)–O(4^{vi}) might be too long, and in both compounds the angle O(2)–O(6)–O(2^{iv}) excludes a hydrogen bond between O(6) and O(2^{iv}). The possible hydrogen bond distances are shown in Fig. 2.

The refinements which were described in Ref. 1 converged to $R=0.100$, 0.089, and 0.087 for the neodymium, gadolinium, and ytterbium compounds, respectively. These values should be compared with the present ones: $\bar{R}=0.061$ for NDG and 0.059 for YDG. The estimated standard deviations are in the range 0.01–0.05 Å for distances and 1–3° for angles in the former case and are about halved in the latter. Even though the standard deviations may be somewhat low (*cf.* p. 3530), the essential aim of this investigation seems to be reached; the effect of the lanthanoid contraction on the tris(oxydiacetato)lanthanoidate group is established.

THE VARIATION OF THE UNIT CELL DIMENSIONS

Approximate unit cell parameters were obtained from Weissenberg and oscillation photographs of the neodymium, gadolinium, and ytterbium oxydiacetate compounds. These parameters were used for a preliminary indexing of the powder photographs of the whole series of lanthanoid oxydiacetates. The unit cell dimensions were improved by least-squares treatments minimizing

$$\sum w(\sin^2\theta_o - \sin^2\theta_c)^2$$

with weights $w = 1/\sin^2 2\theta_o$. For each compound the powder lines were reindexed after each cycle of refinement. The final unit cell parameters and volumes with estimated standard deviations are given in Table 6. The observed values of $\sin^2\theta$ for each compound are compared in Table 7 with those calculated in the last cycle of refinement.

Table 6. The unit cell parameters and volumes with estimated standard deviations of the trigonal compounds $\text{Na}_3[\text{M}(\text{C}_4\text{H}_4\text{O}_6)_3] \cdot 2\text{NaClO}_4 \cdot 6\text{H}_2\text{O}$, $\text{M} = \text{Ce} - \text{Lu}$.

| M | $a/\text{\AA}$ | $c/\text{\AA}$ | $V/\text{\AA}^3$ |
|----|----------------|----------------|------------------|
| Ce | 9.8117(27) | 28.4099(126) | 2368.6(1.3) |
| Pr | 9.7935(15) | 28.3629 (78) | 2355.9(0.7) |
| Nd | 9.7781(13) | 28.3266 (56) | 2345.5(0.5) |
| Sm | 9.7510(16) | 28.2687 (76) | 2327.7(0.8) |
| Eu | 9.7391(18) | 28.2012 (74) | 2316.5(0.9) |
| Gd | 9.7343(17) | 28.1822 (76) | 2312.7(0.8) |
| Tb | 9.7187(18) | 28.1338 (73) | 2301.3(0.9) |
| Dy | 9.7092(12) | 28.1423 (49) | 2297.5(0.6) |
| Ho | 9.7018(15) | 28.1113 (64) | 2291.5(0.7) |
| Er | 9.6988(15) | 28.0973 (60) | 2288.9(0.7) |
| Tm | 9.6940(18) | 28.0759 (65) | 2284.9(0.8) |
| Yb | 9.6832(19) | 28.0599 (74) | 2278.5(0.9) |
| Lu | 9.6787(15) | 28.0428 (59) | 2275.0(0.7) |

The size of the trivalent ions is monotonously decreasing in the lanthanoid series. To prevent "rattling" of the heaviest central ions in the coordination polyhedron the size of the mononuclear tris(oxydiacetato) complex must decrease through the series (*cf.* Table 4.A) and thus influence the unit cell dimensions of the isostructural compounds.

In each unit cell of the structure there are three layers containing the tris(oxydiacetato) complexes. Since these layers are stacked along the c axis the decrease in the cell edge a through the lanthanoid series may be compared with the decrease in $c/3$. In Fig. 3 the quantities

$$p(\text{M}) = r(\text{Ce}) + q(\text{M}) - q(\text{Ce}) \quad (1)$$

for the different lanthanoid oxydiacetates represented by M and with $q = a$ and $c/3$, respectively, are plotted *versus* the set of empirical crystal radii r for the lanthanoid ions determined by Templeton and Dauben.¹⁰

Table 7. Observed and calculated values of $10^4 \cdot \sin^2\theta$ for the compounds $\text{Na}_3[\text{M}(\text{C}_2\text{H}_3\text{O}_5)_3] \cdot 2\text{NaClO}_4 \cdot 6\text{H}_2\text{O}$. The observed powder intensities of the neodymium compound are also given.

| k | Ce | | Pr | | Nd | | Sm | | Eu | | Gd | | Tb | | Dy | | Ho | | Er | | Tm | | Yb | | Lu | | Obs | Nd | | | |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|---|--|
| | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | obs | calc | | | | | |
| 0 3 | 66 | 66 | 67 | 66 | 66 | 67 | 67 | 67 | 67 | 67 | 67 | 67 | 67 | 68 | 67 | 68 | - | 68 | 68 | 68 | 67 | 68 | 67 | 68 | 68 | 68 | 3 | | | | |
| 0 1 | 89 | 90 | 89 | 90 | 90 | 90 | 90 | 91 | 91 | 91 | 91 | 91 | 91 | 91 | 92 | 91 | 92 | 91 | 92 | 91 | 92 | 91 | 92 | 91 | 92 | 91 | 3 | | | | |
| 1 2 | 111 | 112 | 112 | 112 | 112 | 113 | 112 | 113 | 113 | 113 | 113 | 113 | 114 | 113 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 115 | 114 | 115 | 115 | 115 | 3 | | |
| 0 4 | 200 | 200 | 200 | 201 | 201 | 201 | 202 | 202 | 203 | 203 | 203 | 203 | 204 | 204 | 204 | 204 | 204 | 205 | 204 | 205 | 204 | 205 | 204 | 205 | 204 | 205 | 205 | 205 | 3 | | |
| 1 0 | 247 | 247 | 248 | 248 | 248 | 249 | 249 | 250 | 250 | 251 | 251 | 251 | 251 | 252 | 251 | 252 | 252 | 252 | 252 | 252 | 252 | 252 | 252 | 252 | 252 | 254 | 254 | 254 | 3 | | |
| 0 6 | 265 | 265 | 266 | 266 | 266 | 267 | 267 | 268 | 268 | 269 | 269 | 269 | 270 | 267 | 269 | 271 | 270 | 271 | 271 | 271 | 271 | 271 | 271 | 271 | 271 | 272 | 272 | 272 | 3 | | |
| 1 5 | 313 | 313 | 314 | 314 | 315 | 315 | 316 | 317 | 317 | 318 | 318 | 319 | 319 | 319 | 320 | 320 | 320 | 320 | 320 | 321 | 321 | 321 | 321 | 321 | 321 | 321 | 322 | 322 | 3 | | |
| 2 1 | 335 | 337 | 339 | 338 | 339 | 339 | 340 | 341 | 341 | 342 | 344 | 342 | 344 | 343 | 344 | 344 | 344 | 344 | 344 | 345 | 346 | 344 | 345 | 345 | 345 | 346 | 347 | 346 | 3 | | |
| 0 2 | 358 | 359 | 359 | 360 | 361 | 361 | 362 | 363 | 362 | 364 | 367 | 364 | 367 | 366 | 366 | 366 | 366 | 366 | 367 | 367 | 367 | 367 | 367 | 367 | 368 | 368 | 369 | 369 | 3 | | |
| 1 7 | 441 | 443 | 444 | 445 | 446 | 446 | 448 | 448 | 450 | 450 | 452 | 450 | 452 | 452 | 452 | 452 | 452 | 452 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 3 | | |
| 2 4 | 446 | 447 | 449 | 449 | 451 | 450 | 453 | 452 | 454 | 454 | 457 | 454 | 457 | 456 | 456 | 456 | 456 | 456 | 457 | 457 | 457 | 457 | 457 | 457 | 457 | 457 | 457 | 457 | 3 | | |
| 1 6 | 513 | 513 | 514 | 514 | 515 | 515 | 517 | 517 | 518 | 518 | 521 | 517 | 521 | 520 | 522 | 522 | 522 | 522 | 523 | 523 | 523 | 523 | 523 | 523 | 523 | 524 | 524 | 524 | 3 | | |
| 0 5 | 513 | 513 | 515 | 515 | 517 | 517 | 519 | 519 | 521 | 521 | 522 | 522 | 524 | 523 | 524 | 524 | 524 | 525 | 525 | 525 | 525 | 525 | 525 | 525 | 525 | 526 | 527 | 527 | 3 | | |
| 0 9 | 602 | 598 | 598 | 598 | 600 | 600 | 601 | 602 | 603 | 606 | 606 | 606 | 606 | 608 | 608 | 608 | 608 | 609 | 610 | 609 | 610 | 608 | 611 | 610 | 608 | 611 | 610 | 612 | 3 | | |
| 2 2 | 606 | 606 | 608 | 608 | 610 | 610 | 611 | 613 | 614 | 615 | 615 | 615 | 616 | 617 | 618 | 618 | 618 | 619 | 619 | 619 | 620 | 620 | 621 | 621 | 621 | 622 | 622 | 622 | 3 | | |
| 1 7 | 691 | 690 | 694 | 692 | 696 | 694 | 701 | 698 | 698 | 700 | 703 | 701 | 703 | 703 | 704 | 704 | 705 | 705 | 708 | 708 | 708 | 708 | 707 | 707 | 708 | 708 | 709 | 709 | 3 | | |
| 2 4 | 691 | 694 | 697 | 697 | 699 | 699 | 702 | 702 | 704 | 707 | 705 | 707 | 707 | 708 | 708 | 710 | 710 | 710 | 710 | 711 | 711 | 711 | 711 | 712 | 712 | 712 | 713 | 713 | 3 | | |
| 0 0 | 740 | 740 | 741 | 741 | 743 | 743 | 745 | 745 | 747 | 747 | 752 | 752 | 752 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 753 | 3 | | |
| 2 5 | 759 | 760 | 763 | 763 | - | 765 | - | 769 | - | 770 | 772 | - | 772 | 772 | 775 | 775 | 775 | 775 | 776 | 778 | 777 | 778 | 778 | 779 | 778 | 780 | 780 | 781 | 3 | | |
| 0 3 | 806 | 807 | 809 | 810 | 813 | 813 | 816 | 817 | 820 | 820 | 823 | 823 | 824 | 824 | 824 | 824 | 825 | 827 | 826 | 825 | 827 | 826 | 825 | 827 | 826 | 829 | 828 | 829 | 3 | | |
| 1 9 | 842 | 843 | 845 | 846 | 845 | 845 | 852 | 852 | 855 | 855 | 857 | 857 | 861 | 860 | 861 | 860 | 861 | 862 | 862 | 863 | 863 | 863 | 863 | 863 | 863 | 864 | 866 | 866 | 3 | | |
| 1 7 | 935 | 937 | 939 | 940 | 944 | 943 | 948 | 947 | 950 | 951 | 952 | 952 | 957 | 955 | 956 | 956 | 957 | 958 | 957 | 958 | 958 | 959 | 959 | 959 | 959 | 961 | 962 | 961 | 3 | | |
| 1 11 | 971 | 973 | 976 | 977 | 977 | 979 | 981 | 983 | 985 | 988 | 990 | 989 | 993 | 992 | 993 | 992 | 993 | 994 | 995 | 995 | 995 | 995 | 997 | 999 | 999 | 999 | 999 | 999 | 3 | | |
| 2 0 | 986 | 988 | 991 | 991 | 994 | 995 | 998 | 1000 | 1002 | 1002 | 1004 | 1004 | 1008 | 1007 | 1009 | 1009 | 1009 | 1010 | 1010 | 1011 | 1011 | 1011 | 1011 | 1011 | 1011 | 1012 | 1014 | 1015 | 1015 | 3 | |
| 2 8 | 1004 | 1006 | 1008 | 1010 | 1011 | 1011 | 1018 | 1018 | 1020 | 1021 | 1021 | 1022 | 1024 | 1025 | 1028 | 1027 | 1028 | 1028 | 1031 | 1029 | 1031 | 1030 | 1031 | 1030 | 1031 | 1032 | 1032 | 1033 | 3 | | |
| 0 6 | 1052 | 1047 | - | 1051 | - | 1054 | - | 1059 | - | 1063 | - | 1064 | 1067 | 1068 | 1068 | 1069 | 1070 | 1071 | 1072 | 1071 | 1072 | 1071 | 1072 | 1071 | 1072 | 1071 | 1075 | 1075 | 1075 | 3 | |
| 2 3 | 1054 | 1054 | - | 1058 | - | 1061 | - | 1067 | - | 1070 | - | 1070 | 1071 | 1074 | 1074 | 1076 | 1076 | 1076 | 1076 | 1078 | 1081 | 1079 | 1082 | 1080 | 1081 | 1082 | 1083 | 1083 | 3 | | |
| 0 12 | - | 1060 | - | 1064 | - | 1067 | - | 1071 | - | 1076 | - | 1077 | - | 1081 | - | 1081 | - | 1081 | - | 1083 | 1084 | 1086 | 1086 | 1087 | 1087 | 1088 | 1088 | 1089 | 3 | | |
| 2 10 | - | 1066 | 1071 | 1069 | 1072 | 1072 | 1079 | 1077 | 1082 | 1081 | 1082 | 1083 | 1083 | 1083 | 1086 | 1086 | 1087 | 1086 | 1089 | 1091 | 1090 | 1091 | 1091 | 1091 | 1094 | 1093 | 1092 | 1094 | 3 | | |
| 3 1 | 1078 | 1077 | 1083 | 1081 | 1083 | 1085 | 1089 | 1091 | 1093 | 1096 | 1095 | 1099 | 1098 | 1100 | 1100 | 1102 | 1102 | 1103 | 1102 | 1107 | 1104 | 1107 | 1104 | 1107 | 1106 | 1106 | 1107 | 1107 | 3 | | |
| 1 2 | 1099 | 2000 | 1102 | 1104 | 1106 | 1108 | 1110 | 1111 | 1115 | 1116 | 1117 | 1117 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 1121 | 3 | | |
| 3 4 | 1191 | 1188 | 1194 | 1192 | 1194 | 1196 | 1203 | 1202 | 1207 | 1206 | 1209 | 1207 | 1211 | 1211 | 1211 | 1211 | 1211 | 1212 | 1215 | 1218 | 1216 | 1219 | 1217 | 1220 | 1219 | 1221 | 1221 | 1221 | 3 | | |
| 0 11 | 1221 | 1220 | 1227 | 1224 | 1226 | 1228 | 1233 | 1233 | 1238 | 1238 | 1243 | 1240 | 1248 | 1244 | 1244 | 1244 | 1244 | 1249 | 1247 | 1247 | 1248 | 1251 | 1250 | 1252 | 1251 | 1254 | 1252 | 1252 | 3 | | |
| 2 6 | 1234 | 1234 | 1235 | 1235 | 1236 | 1238 | 1243 | 1243 | 1249 | 1249 | 1274 | 1274 | 1277 | 1281 | 1280 | 1286 | 1286 | 1287 | 1287 | 1288 | 1288 | 1288 | 1288 | 1288 | 1288 | 1288 | 1288 | 1288 | 3 | | |
| 1 12 | 1308 | 1307 | - | 1312 | 1311 | 1315 | 1320 | 1321 | 1328 | 1327 | 1325 | 1328 | 1330 | 1333 | 1333 | 1333 | 1334 | 1336 | 1339 | 1337 | 1340 | 1339 | 1341 | 1341 | 1341 | 1343 | 1342 | 1342 | 3 | | |
| 1 10 | 1313 | 1313 | 1316 | 1317 | 1320 | 1320 | 1328 | 1327 | 1332 | 1332 | 1336 | 1336 | 1336 | 1337 | 1338 | 1339 | 1341 | 1341 | 1342 | 1343 | 1344 | 1344 | 1344 | 1344 | 1344 | 1348 | 1348 | 1348 | 3 | | |
| 0 1 | 1325 | 1324 | 1333 | 1332 | 1335 | 1333 | 1337 | 1341 | 1342 | 1344 | 1342 | 1345 | 1351 | 1350 | 1350 | 1352 | 1353 | 1354 | 1356 | 1355 | 1363 | 1357 | 1359 | 1361 | 1360 | 1361 | 1361 | 1361 | 3 | | |
| 0 13 | 1325 | 1327 | 1334 | 1333 | 1335 | 1335 | 1340 | 1347 | 1346 | 1348 | 1348 | 1348 | 1351 | 1353 | 1353 | 1352 | 1354 | 1355 | 1355 | 1356 | 1356 | 1356 | 1356 | 1357 | 1357 | 1358 | 1358 | 1358 | 3 | | |
| 0 9 | 1342 | 1342 | 1344 | 1342 | 1347 | 1346 | 1353 | 1352 | 1359 | 1358 | 1366 | 1366 | 1364 | 1366 | 1366 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 1367 | 3 | | |
| 2 4 | 1350 | 1346 | 1354 | 1351 | 1356 | 1356 | 1363 | 1363 | 1366 | 1367 | 1366 | 1368 | 1369 | 1372 | 1376 | 1375 | 1377 | 1377 | 1378 | 1378 | 1378 | 1378 | 1378 | 1378 | 1378 | 1378 | 1378 | 1378 | 3 | | |
| 3 7 | 1435 | 1431 | 1436 | 1436 | 1439 | 1440 | 1445 | 1448 | 1455 | 1452 | 1456 | 1453 | 1457 | 1459 | 1460 | 1460 | 1462 | 1463 | 1463 | 1464 | 1464 | 1466 | 1466 | 1467 | 1468 | 1468 | 1470 | 1470 | 3 | | |
| 2 4 | 1435 | 1435 | 1440 | 1444 | 1445 | 1454 | 1452 | 1456 | 1460 | 1458 | 1463 | 1462 | 1466 | 1465 | 1467 | 1467 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 1468 | 3 | | |
| 1 11 | 1468 | 1467 | 1472 | 1472 | 1476 | 1476 | 1484 | 1484 | 1490 | 1490 | 1489 | 1491 | 1495 | 1496 | 1497 | 1496 | 1498 | 1499 | 1501 | 1501 | 1502 | 1503 | 1507 | 1505 | 1509 | 1507 | 1509 | 1509 | 3 | | |
| 1 14 | 1525 | 1530 | 1531 | 1537 | 1535 | 1543 | 1540 | 1550 | 1548 | 1550 | 1550 | 1553 | 1555 | 1555 | 1555 | 1555 | 1559 | 1558 | 1559 | 1560 | 1561 | 1562 | 1562 | 1564 | 1564 | 1566 | 1566 | 3 | | | |
| 1 8 | 1540 | 1541 | 1543 | 1547 | - | 1551 | 1560 | 1559 | 1566 | 1564 | 1566 | 1566 | - | 1571 | 1572 | 1573 | 1575 | 1576 | 1576 | 1577 | 1580 | 1578 | 1582 | 1582 | 1584 | 1581 | 1581 | 3 | | | |
| 2 1 | 1576 | 1571 | 1580 | 1577 | 1581 | 1583 | 1583 | 1591 | 1597 | 1594 | 1596 | 1596 | 1601 | 1605 | 1610 | 1607 | 1605 | 1610 | 1608 | 1609 | 1611 | 1611 | 1612 | 1 | | | | | | | |

Table 8. Values of $\sqrt[3]{(V/Z)}$ for the two series of compounds $\text{Na}_3[\text{M}(\text{C}_4\text{H}_4\text{O}_5)_3] \cdot 2\text{NaClO}_3 \cdot 6\text{H}_2\text{O}$ (denoted O) and $[\text{M}(\text{H}_2\text{O})_9](\text{BrO}_3)_3$ (B)¹³ compared with the crystal radii (r) of the trivalent lanthanoid ions.¹⁰

| M | $\frac{\sqrt[3]{(V/Z)}/\text{\AA}}{O}$ | $\frac{\sqrt[3]{(V/Z)}/\text{\AA}}{B}$ | $r/\text{\AA}$ | M | $\frac{\sqrt[3]{(V/Z)}/\text{\AA}}{O}$ | $\frac{\sqrt[3]{(V/Z)}/\text{\AA}}{B}$ | $r/\text{\AA}$ |
|----|--|--|----------------|----|--|--|----------------|
| Ce | 9.242 | 7.44 | 1.034 | Tb | 9.154 | 7.37 | 0.932 |
| Pr | 9.226 | 7.43 | 1.013 | Dy | 9.149 | 7.36 | 0.908 |
| Nd | 9.213 | 7.42 | 0.995 | Ho | 9.141 | 7.34 | 0.984 |
| Pm | — | — | 0.979 | Er | 9.138 | 7.33 | 0.881 |
| Sm | 9.189 | 7.40 | 0.964 | Tm | 9.132 | 7.33 | 0.869 |
| Eu | 9.174 | 7.39 | 0.950 | Yb | 9.124 | 7.33 | 0.858 |
| Gd | 9.169 | 7.38 | 0.938 | Lu | 9.119 | 7.32 | 0.848 |

$\sigma \approx 0.002$ for the oxydiacetates and 0.01 for the bromates.

in the two series of compounds. Mayer and Glasner¹³ have determined the unit cell dimensions of the lanthanoid bromates. Their data were obtained with Ni-filtered Cu radiation using a powder diffractometer. Since the decreases in a and c are rather parallel it seems reasonable to calculate values of $\sqrt[3]{(V/Z)}$ for the bromate compounds. These values are given in Table 8 too.

The contraction of the oxydiacetate and bromate compounds are compared with the contraction of the trivalent ions Fig. 4. The quantities p of eqn. (1) with $q = \sqrt[3]{(V/Z)}$ are plotted *versus* the crystal radii for both series. The oxydiacetates are divided in two groups. The decrease in $\sqrt[3]{(V/Z)}$ is slightly less than

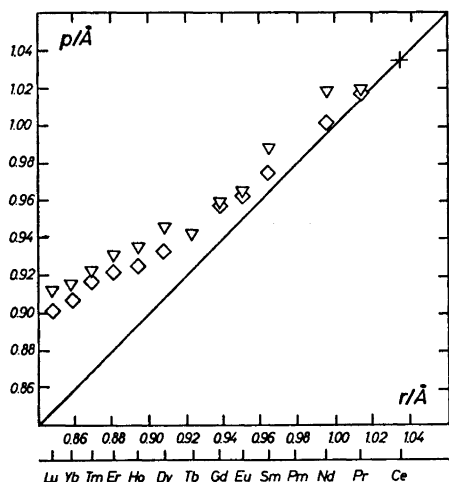


Fig. 3. Values of p of eqn. (1) with $q = a$ (\diamond) and $c/3$ (∇) plotted *versus* the crystal radius, r , of the trivalent lanthanoid ions. For comparison the line $p = r$ is included.

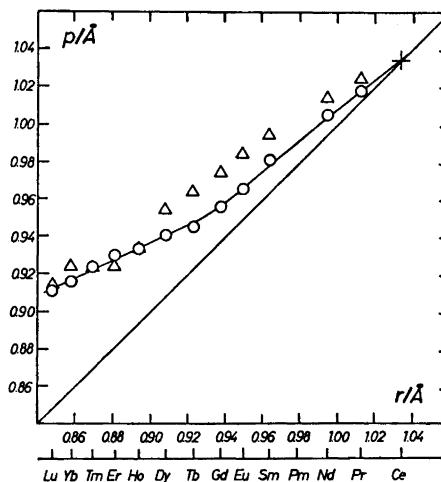


Fig. 4. Values of p with $q = \sqrt[3]{(V/Z)}$ for the oxydiacetate (O) and bromate (\triangle) compounds plotted *versus* r . For comparison the line $p = r$ is included.

the decrease in crystal radius of the ions before gadolinium but is appreciably smaller than that for the heaviest ions. Taking the error in $\sqrt[3]{(V/Z)}$ into account one might conclude that the bromates behave as the oxydiacetates.

THE EFFECT OF THE LANTHANOID CONTRACTION

The coordination polyhedron around the lanthanoid ion in the tris(oxydiacetato)complex shrinks 0.133 ± 0.015 Å along the *c* axis and 0.137 ± 0.018 Å along the *a* and *b* axes between NDG and YDG. The differences in the M–O(1) and M–O(2) bond distances are 0.091 ± 0.013 Å and 0.089 ± 0.008 Å, respectively. These latter values are appreciably less than the decrease in crystal radius, 0.137 Å. The distances between adjacent oxygen atoms coordinated to the lanthanoid ion but not belonging to the same ligand *i.e.* O(1)–O(2ⁱⁱⁱ) and O(2)–O(2ⁱⁱ), have in YDG values compatible with twice the van der Waals radius of oxygen.¹⁴ They are 2.851 Å and 2.924 Å, respectively. In NDG the corresponding lengths are both 3.026 Å.

In order to describe the contraction of the outer parts of the tris(oxydiacetato) complex it seems reasonable to look at the carboxylate oxygens O(3). They form a slightly irregular octahedron around the lanthanoid ion with edges in the interval 6.0–7.1 Å (see Fig. 1). The dimensions of this octahedron decrease 0.144 ± 0.017 Å along the *c* axis and 0.210 ± 0.017 Å along the *a* and *b* axes between NDG and YDG. The difference in the contraction of the coordination polyhedron and the O(3)-octahedron is, if real, a result of a change in the structure of the ligand between the two compounds (*cf.* p. 3535).

When the decrease in the dimensions of the complex ion is compared with the corresponding difference in $\sqrt[3]{(V/Z)}$, 0.089 ± 0.003 Å, it is found that the decrease in $\sqrt[3]{(V/Z)}$ is not a direct measure of the contraction of the tris(oxydiacetato)lanthanoidate group. The layer containing the sodium ions and water molecules seems thus to partly obstruct the contraction imposed on the structure by the shrinking complex ion. This is corroborated by the distances Na(1)–Na(2), Na(1)–O(6), and Na(2)–O(6), which are almost unchanged between NDG and YDG as is seen in Table 4. C.

In view of the present single crystal measurements on NDG and YDG one may describe the trend in $\sqrt[3]{(V/Z)}$ in the following way. With lanthanoid ions lighter than gadolinium the repulsive forces between the coordinated oxygen atoms of the tris(oxydiacetato) complex are small and the contraction of the complex ion is opposed mainly by the other parts of the structure. The trend thus imposed on $\sqrt[3]{(V/Z)}$ is reinforced for the heaviest central ions by van der Waals repulsions between the coordinated oxygen atoms, resulting in a changed slope of $p(M)$ in the region Gd–Dy as is seen in Fig. 4.

A further conclusion that might be reached is that the similarity between the bromate and oxydiacetate compounds shown in Fig. 4 does not necessarily depend on a similar contraction of the two coordination polyhedra through the lanthanoid series; it might as well be a result of small changes in other parts of the structure of the bromate compounds counteracting the contraction of the nona-aquo complex.

The thermodynamic data for the oxydiacetate complexes of the rare earth ions in water solution have been determined by Grenthe¹⁵ and by Grenthe and

Hansson.¹⁶ Their measurements indicate among other things increasing steric hindrance for the formation of the third complex with decreasing radius of the central ion. As the mononuclear tris(oxydiacetato) complexes are rather isolated and thus exposed only to weak interactions in the solid compounds $\text{Na}_3[\text{M}(\text{C}_4\text{H}_4\text{O}_5)_3] \cdot 2\text{NaClO}_4 \cdot 6\text{H}_2\text{O}$, it is very probable that these complexes have almost the same structure in the solid state as in solution. Therefore, the result of the present investigation supports Grenthe and Hansson's interpretation of the thermodynamic solution data.

Added in proof. The unit cell dimensions of the bromate compounds $[\text{M}(\text{H}_2\text{O})_9](\text{BrO}_3)_3$, $\text{M} = \text{La} - \text{Lu}$, have been reinvestigated using Guinier powder photographs.¹⁷ It is found that $(V/Z)^{1/3}$ is approximately a linear function of r and that the decrease in it is about 0.135 Å between the cerium and lutetium compounds.

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